

Draft Syllabus

M.Sc. in Mathematics

[Choice Based Credit System (CBCS)]

Faculty of Science

(School of Physical Sciences)

Department of Mathematics

MAHATMA GANDHI UNIVERSITY

PURBA MEDINIPUR-721628

West Bengal

(w.e.f. Academic Session 2021-2022)

Course Structure of M.Sc. in Mathematics

Semester	Course Code	Name of the Course	Full Marks	Total Class Hours	Credit
Semester-I	MATH.101	Abstract Algebra	50	50	5
	MATH.102	Linear Algebra and Modules	50	50	5
	MATH.103	Real Analysis	50	50	5
	MATH.104	Ordinary Differential Equations and Special Functions.	50	50	5
	MATH.105	Classical Mechanics	50	50	5
Semester-II	MATH.201	Complex Analysis	50	50	5
	MATH.202	General Topology	50	50	5
	MATH.203	Numerical Analysis	50	50	5
	MATH.204	Partial Differential Equations and Generalized Functions	50	50	5
	MATH.205 (CBCS)	History of Mathematics	50	50	4
Semester-III	MATH.301	General Theory of Continuum Mechanics	50	50	5
	MATH.302	UNIT-I: Stochastic Process & Statistics UNIT-II: MATLAB Programming (Practical)	25 25	50	5
	MATH.303	Special Paper: Operations Research – I / Advanced Algebra – I / Mathematical Modelling In The Biological Environment – I	50	50	5
	MATH.304	Special Paper: Operations Research – II / Advanced Algebra – II / Mathematical Modelling In The Biological Environment – II	50	50	5
	MATH.305 (CBCS)	Statistical Methods using MATLAB	50	50	4
Semester-IV	MATH.401	Functional Analysis	50	50	5
	MATH.402	Mathematical Methods	50	50	5
	MATH.403	Special Paper: Operations Research – III / Advanced Algebra – III / Mathematical Modelling In The Biological Environment – III	50	50	5
	MATH.404	Special Paper: Operations Research – IV / Advanced Algebra – IV / Mathematical Modelling In The Biological Environment – IV	50	50	5
	MATH.405	Social Outreach Programme DISSERTATION PROJECT WORK	10 40	50	5
		Total	1000	1000	98

	Semester-I			
Course Code	Name of the Course	Full Marks	Total Class Hours	Credit
MATH.101	Abstract Algebra	50	50	5
MATH.102	Linear Algebra and Modules	50	50	5
MATH.103	Real Analysis	50	50	5
MATH.104	Ordinary Differential Equations and Special Functions.	50	50	5
MATH.105	Classical Mechanics	50	50	5
	Total	250	250	25

Paper: MATH.101

Abstract Algebra

Total Lectures: 50

Full Marks: 50

Groups : Direct product and semi-direct product of groups, automorphisms and inner automorphisms, conjugacy class equation and consequences, group actions, applications of group actions and generalized Cayley theorem, simple groups, p-groups, Cauchy's theorem, Converse of Lagrange's theorem for finite abelian groups, Sylow's theorems and their applications, classifications of finite groups of order up to 15, normal and subnormal series, composition series, solvable groups and nilpotent groups, Jordan-Hölder theorem and its applications.

Rings : Prime and maximal ideals, field of quotient of an integral domain, polynomial rings, divisibility theory in integral domain, Euclidean Domain, Principal Ideal Domain, irreducible and prime element, Unique Factorization Domain, Eisenstein's irreducibility criterion.

Fields : Algebraic and transcendental extensions, finite extension, algebraically closed field, splitting field, separable extension, impossibility of some constructions by straightedge and compass, finite fields and their properties, normal extension, Galois group of automorphisms, Galois theory, solution of polynomial equations by radicals, insolvability of the general equation of degree 5 (or more) by radicals.

Texts / References :

1. David S. Dummit, Richard M. Foote, *Abstract Algebra*, 3rd edition, John Wiley & Sons, 2003.
2. J. B. Fraleigh, *A first course in Abstract algebra*, 7th edition, Addison-Wesley, 2003.
3. Joseph A Gallian, *Contemporary Abstract Algebra*, Cengage Learning India Pvt. Ltd., 2019.
4. P. B. Bhattacharya, S. K. Jain and S. R. Nagpal, *Basic Abstract Algebra*, Second Edition, Cambridge.
5. T. W. Hungerford, *Abstract Algebra, An Introduction*, 3rd edition, Brooks/Cole Cengage Learning.
6. D. S. Malik, J. N. Mordeson and M. K. Sen, *Fundamentals of Abstract Algebra*, McGraw Hill.
7. Herstein, I. N., *Topics in Abstract Algebra*, Wiley Eastern Limited.
8. Hungerford, T.W., *Algebra*, Springer.
9. Goldhaber, J.K., Ehrlich, G., *Algebra*, The Macmillan Company, Collier-Macmillan Limited, London.
10. Jacobson, N., *Basic Algebra*, I & II, Hindustan Publishing Corporation, India.
11. Rotman, J.J., *The Theory of Groups: An Introduction*, Allyn and Bacon, Inc., Boston.

Paper: MATH.102

Linear Algebra and Modules

Total Lectures: 50

Full Marks: 50

Linear functional, dual space and dual basis, double dual and related results, transpose of a linear transformation and matrix representation of the transpose of a linear transformation.

Characteristic and minimal polynomial, similarity of linear operator, invariant sub-spaces, primary decomposition theorem, diagonalizable form, triangular canonical form, idempotent operator, nilpotent operator, generalised eigen vectors, Jordan blocks and Jordan canonical form, rational canonical form.

Linear transformations on inner product spaces, adjoint of a linear operator, matrix representation of the adjoint, normal and self-adjoint operators, unitary and orthogonal operators and their matrices, orthogonal projections, spectral theorems and consequences.

Bilinear forms, symmetric and skew-symmetric bilinear forms, real quadratic forms, Sylvester's law of inertia, positive definiteness.

Module, sub module, quotient module, module homomorphism, isomorphism theorems, cyclic module, simple module, free module, finitely generated free module, torsion module, torsion free module, finitely generated module over PID.

Texts / References :

1. *Kenneth Hoffman and Ray Kunze, Linear Algebra, Prentice-Hall International, 2010.*
2. *P. Lax, Linear Algebra, John Wiley & Sons, 1997.*
3. *H. E. Rose, Linear Algebra, Birkhauser, 2002.*
4. *S. Lang, Algebra, 3rd Edition, Springer (India), 2004.*
5. *Roger A. Horn, Charles R. Johnson., Matrix Analysis, 2nd edition, CUP, 2013.*
6. *Sheldon Axler., Linear Algebra Done Right, 3rd edition, Springer, 2015.*
7. *N. Jacobson., Basic algebra I, 2nd edition, Dover Publications, 2012.*
8. *N. Jacobson., Basic algebra II, 2nd edition, Dover Publications, 2012.*
9. *T. S. Blyth., Module Theory : An approach to Linear Algebra., Clarendon Press, Oxford, 1977.*
10. *S. Roman, Advanced Linear Algebra, 3rd edition, Springer, 2007.*
11. *J. H. Kwak and S. Hong, Linear Algebra, 2nd edition, Birkhauser, 2006.*

Paper: MATH.103

Real Analysis

Total Lectures: 50

Full Marks: 50

Functions of Bounded Variation : Definitions, basic properties and geometrical meaning of functions of bounded variation, Differentiation of a function of bounded variation, Representation of an absolutely continuous function by an integral, Total variation, Jordan's theorem.

The Riemann-Stieltjes Integral : The definition of Riemann-Stieltjes integral, Linear properties, Reduction to a Riemann integral, Reduction of a Riemann-Stieltjes integral to a finite sum, Euler's summation formula, Upper and lower integrals, Riemann's condition, Sufficient conditions for the existence of Riemann-Stieltjes integrals.

Lebesgue Measure : Measurable Sets, Concept of Lebesgue function, Inner and outer measure. It's simple properties. Set of measure zero. Cantor set, Borel set and their measurability, Non-measurable sets, Lebesgue measure, Measure space.

Measurable function : Definition and it's simple properties, Borel measurable functions, sequence of measurable functions, Statement of Lusin's theorem, Egoroff's theorem. Simple functions and it's properties.

Lebesgue integral on a measurable set : Definition, Basic simple properties, Lebesgue integral of a bounded function over a set of finite measure. Simple properties. Integral of nonnegative measurable functions, General Lebesgue integral. Bounded convergence theorem for a sequence of Lebesgue integrable function, Fatou's lemma. Classical Lebesgue dominated convergence theorem. Monotone convergence theorem, Relation between Lebesgue integral and Riemann integral.

Texts / References :

1. W. Rudin., *Principles of mathematical Analysis*, McGraw-Hill Publishing Company, 2006.
2. Robert G. Bartle., *The elements of Real Analysis*, Wiley, New York, 1964.
3. T. M. Apostol., *Mathematical Analysis*, 2nd edition, Narosa Book Distributors Pvt Ltd-New Delhi, 1996.
4. Ghorpade & Limyae., *A Course in Multivariable Calculus and Analysis*, 2nd edition, Springer, 2009.
5. Ajit Kumar, S. Kumaresan. *A Basic Course in Real Analysis*, 1st edition, CRC press, 2014.
6. G. De Barra., *Introduction to Measure Theory*, 3rd edition, Van Nostrand Reinhold Company Ltd, 1974.
7. Inder K. Rana., *An Introduction to Measure and Integration*, 2nd edition, Narosa, 2007.

Paper: MATH.104

Ordinary Differential Equations and Special Functions

Total Lectures: 50

Full Marks: 50

First order system of equations: Well-posed problems, Existence and uniqueness of solutions of initial value problems, simple illustrations, Peano's and Picard's theorems (statements only).

Linear systems, non-linear autonomous system, phase plane analysis, critical points, stability, Linearization, Liapunov stability, undamped pendulum, Applications to biological system and ecological system.

General theory of homogenous and non-homogeneous linear ODEs, Two-point boundary value problem for a second order linear non homogeneous O.D.E.

Eigen Value Problem: Ordinary differential equations of the Sturm-Liouville type, Properties of Sturm Liouville type, Application to Boundary Value Problems, Eigen values and Eigen functions, Orthogonality theorem, Expansion theorem.

Green's Function: Green's Function and its properties, Green's function for ordinary differential equations, Application to Boundary Value Problems.

Dynamical systems: A brief history of Dynamical systems, Continuous and discrete systems, Flows and maps, fixed points, stability, Qualitative analysis of one-dimensional flows, periodic points and their stabilities, Different attractors, ω - and α - limit points, wandering point, Basin of attraction.

Series solution: Ordinary and singular points of a second order linear differential equation in the complex plane; Fuchs's theorem, series solution about an ordinary point with examples, Regular singularity, series solution about regular singular point with examples, Frobenius' method,

Legendre equation: Series solution, Legendre polynomial, Legendre functions of first kind and second kind, Laplace integral, Generating function, Rodrigue's formula, Recurrence relations, orthogonal properties of Legendre polynomials, Schlaefli's integral.

Bessel equation: Series solution of Bessel equation, Bessel function, Generating function, Recurrence relations, Integral representations of Bessel's functions, asymptotic expansion of Bessel functions.

Texts / References:

1. E. A. Coddington and Levinson – *Theory of Ordinary Differential Equations* [TMH].
2. R.P. Agarwal & R. C. Gupta – *Essentials of Ordinary Differential Equations* [MGH]
3. G.F. Simmons: *Differential Equations*, TMH Edition, New Delhi, 1974.
4. S.L. Ross: *Differential Equations* (3rd edition), John Wiley & Sons, New York, 1984.
5. J. C. Burkill – *The Theory of Ordinary Differential Equations* [Oliver & Boyd, London].
6. G. Birkhoff & G. Rota - *Ordinary Differential Equation* [Ginn].
7. E. D. Rainville – *Special Functions* [Macmillan].
8. I. N. Sneddon - *Special Functions of mathematical Physics & Chemistry* [Oliver & Boyd, London].
9. N. N. Lebedev - *Special Functions and Their Applications* [PH].

Paper: MATH.105

Classical Mechanics

Total Lectures: 50

Full Marks: 50

Rotating Frame of Reference: Rotating coordinate system, Equation of motion of a free particle relative to the rotating earth, Effects of earth's rotation.

Constrained Motion: Constraints and its classification, Constraint forces, Dynamical system, Generalized coordinates, Classification of dynamical system, Degrees of freedom, D'Alembert's Principle, Lagrange's equation of motion of first kind, Principle of virtual work. Plane pendulum and spherical pendulum. Cyclic co-ordinates. Coriolis force.

Lagrangian Mechanics: Lagrange's equation of motion, Lagrange's equation of motion for anonholonomic dynamical system, Rayleigh's dissipation function, Physical interpretation of the dissipative function.

Hamiltonian Mechanics: Ignorable coordinates, Noether's theorem, Routh's process for the ignoring of coordinates; Hamiltonian of a dynamical system: Conservation laws. Infinitesimal transformations, Space-time transformations. Conservation theorem for angular momentum, Condition of Invariance, Legendre dual transformations, Hamiltonian of a system; Hamilton's equations of motion, Applications of Hamiltonian mechanics.

Variational Principles: Variation of a functional, Hamilton's principle, Establishment of Hamilton's principle from D'Alembert's principle, Deduction of Lagrange's equations of motion from Hamilton's principle, Deduction of Hamilton's equations of motion from Hamilton's principle, Extended Hamilton's principle, Deduction of Lagrange's equations of motion for non-conservative dynamical system, Principle of least action, Brachitochrone problem,

Canonical Transformations: Aspects of canonical transformation, Generating function, Principal forms of generating function, Condition for canonicity, Contact transformation. Liouville's theorem.

Brackets: Poisson brackets, Poisson theorem, Hamilton's equations of motion in terms of Poisson's brackets, Lagrange brackets, Properties of Poisson brackets, Constants of the motion, Hamilton-Jacobi equation.

Calculus of variations: Euler-Lagrange equation, Applications of Euler-Lagrange equation, Isoperimetric problems.

Motion of a Rigid Body: Motion of a system of particles, Small oscillations of a conservative dynamical system with one degree of freedom, Small oscillations of a conservative dynamical system with n degrees of freedom, Normal coordinates, Euler angles, Linear and angular momentum of a rigid body, Kinetic energy of a rigid body, Equation of motion of a rigid body, Euler's equations of motion of a rigid body about a fixed point and Euler's equations of motion of a rigid body about a fixed axis, Motion of a top, Steady motion of a top, Stability of top motion

Special theory of relativity in Classical Mechanics:-Postulates of special relativity. Applications of Classical Mechanics in Special Theory of Relativity: Lorentz transformation, Applications of Lorentz transformation, Velocity and acceleration in relativistic mechanics, Force and energy in relativistic mechanics.

Texts / References:

1. *H. Goldstein, Classical Mechanics, Addison-Wesley, Cambridge, 1950.*
2. *A. S. Gupta, Calculus of Variations with Applications, Prentice-Hall of India, New Delhi, 2005.*
3. *B. D. Gupta and S. Prakash, Classical Mechanics, KedarNath Ram Nath, Meerut, 1985.*
4. *T.W.B. Kibble, Classical Mechanics, Orient Longman, London, 1985.*
5. *N. C. Rana and P. S. Joag, Classical Mechanics, Tata McGraw-Hill Publishing Company Limited, New Delhi, 2004.*
6. *A. K. Raychaudhuri, Classical Mechanics-A Course of Lectures, Oxford University Press, Calcutta, 1983.*
7. *M. R. Spiegel, Theoretical Mechanics, Schaum Series, New York, 1967.*
8. *K. R. Symon, Mechanics, Addison-Wesley Publ. Co., Inc., Massachusetts, 1971.*
9. *R. G. Takwale and S. Puranik, Introduction to Classical Mechanics, Tata McGraw-Hill Publ. Co. Ltd., New Delhi, 1980.*

Course Structure of M.Sc. in Mathematics

Semester-II				
Course Code	Name of the Course	Full Marks	Total Class Hours	Credit
MATH.201	Complex Analysis	50	50	5
MATH.202	General Topology	50	50	5
MATH.203	Numerical Analysis	50	50	5
MATH.204	Partial Differential Equations and Generalised Functions	50	50	5
MATH.205	History of Mathematics	50	50	4
	Total	250	250	24

Paper: MATH.201

Complex Analysis

Total Lectures: 50

Full Marks: 50

Analytic functions, Zeroes of Analytic functions, Cauchy-Goursat Theorem, Homotopy version of Cauchy's theorem, Morera's theorem, uniqueness and maximum principle and its applications.

Singularity and its classification, singularity at infinity, behaviour of function near isolated singularity, Riemann's theorem, limit points of zeroes and poles, Casorati – Weierstrass's theorem, multivalued functions – branch point, idea of winding number, Analytic continuation, Monodromy theorem.

Residue at a finite point, Residue at Infinity, the residue theorem, integral over the real axis, improper integrals and principal values, integrands with branch points, Principle of the argument, Rouché's theorem, open mapping theorem, deformation of contours, Schwarz's lemma.

Conformal mapping, bilinear transformation, harmonic functions and mappings, the Schwarz – Christoffel transformation, Hurwitz polynomials and positive functions, Inverse mappings and univalent functions, Global mapping theorems, the Riemann mapping theorem.

Convergence of sequences, convergence of series, Formulas of Parseval, Schwarz and Poisson, Asymptotic series.

Texts / References:

1. Complex Analysis, T W Gamelin, Springer.
2. Real and Complex Analysis, W. Rudin, Mc Graw Hill Private Ltd.
3. Complex Analysis: The Geometric Viewpoint, S G Krantz, JSTOR
4. Complex Variable and Applications, J. W. Brown and R. V. Churchill, 8th Edition, GcGraw Hill.
5. Foundations of Complex Analysis, S. Ponnusamy, Narosa, 1995.
6. Complex Variables, N. Levinson and R.M. Redheffer, McGraw Hill Education (India) Private Limited, 2015

Paper: MATH.202

General Topology

Total Lectures: 50

Full Marks: 50

Topological spaces and Continuous Functions : Topology on a set, Examples of Topologies (Topological Spaces): Discrete Topology, Indiscrete Topology, Finite Complement Topology, Countable Complement Topology, Topologies on the Real Line, Finer and Coarser Topologies, Basis and Sub basis for a topology, Product topology on $X \times Y$, Subspace Topology, Metric topology, Order Topology.

Interior Points, Limit Points, Derived Set, Boundary of a set, Closed Sets, Closure and Interior of a set, Kuratowski closure operator and the generated topology.

Continuous Functions, Rules for Constructing Continuous Functions : Inclusion Map, Composition, by restricting the domain, by restricting/expanding the range, Pasting Lemma, Open maps, Closed maps and Homeomorphisms.

Product Topology : Sub basis for product Topology defined by Projection Maps, Box Topology, Quotient Topology, Metric Topology.

Connectedness and Compactness : Connected and Path connected spaces, Connected subspaces of the real line, Components and local connectedness, Compact spaces, Heine-Borel Theorem, Local-compactness, Tychonoff's Theorem on compact spaces.

Separation Axioms : 1st and 2nd countable spaces, Hausdorff spaces, Regularity, Complete irregularity, normality, Hausdorff spaces, Regularity, Complete Regularity, Normality, Urysohn Lemma, Tychonoff embedding and Urysohn Metrization Theorem, Tietze Extension Theorem (Statement Only).

Texts / References :

1. J. R. Munkres, *Topology*, 2nd Ed., Pearson Education (India), 2000.
2. M. A. Armstrong, *Basic Topology*, Springer (India), 1983.
3. K. D. Joshi, *Introduction to General Topology*, New Age International Private Limited, New Delhi, 2014.
4. G. F. Simmons, *Introduction to Topology and Modern Analysis*, McGraw-Hill, New York, 1963.
5. W. J. Thron – Topological Structures.
6. K. D. Joshi – Introduction to General Topology.
7. J. L. Kelly – General Topology.
8. G. F. Simmons – Introduction to Topology & Modern Analysis.

Paper: MATH.203

Numerical Analysis

Total Lectures: 50

Full Marks: 50

Cubic spline interpolation, Lagrange's bivariate interpolation, Approximation of function, Chebyshev polynomial: Minimax property, Curve fitting by least square method, Use of orthogonal polynomials, Economization of power series.

Numerical integration: Newton-Cotes formulae-open type, Gaussian quadrature: Gauss-Legendre, Gauss-Chebyshev, Integration by Monte Carlo method.

Roots of polynomial equation: Bairstow method.

System of linear equations: Pivoting, Matrix inverse, LU decomposition method, Tri-diagonal system of equations, Ill-conditioned linear systems, Relaxation method.

System of non-linear equations, fixed point method,, Newton methods, Convergence, rate of convergence.

Eigen value problem, Power method, Jacobi's method.

Ordinary differential equation: Runge-Kutta method for linear ODEs and second order IVP, Predictor-corrector method: Milne's method, Stability, Second order BVP: Shooting method, finite difference method, finite element method.

Partial differential equation: Finite difference scheme, Explicit and implicit methods of Hyperbolic and Parabolic equations, Crank-Nicolson method, Elliptic equation, Stability, Consistency and convergence.

Texts / References:

1. A. Gupta and S.C. Bose, *Introduction to Numerical Analysis*, Academic Publishers, Calcutta, 1989.
2. M.K. Jain, S.R.K. Iyengar and R.K. Jain, *Numerical Methods for Scientific and Engineering Computation*, New Age International (P) Limited, New Delhi, 1984.
3. E.V. Krishnamurthy and S.K. Sen, *Numerical Algorithms*, Affiliated East-West Press Pvt. Ltd., New Delhi, 1986.
4. J.H. Mathews, *Numeical Methods for Mathematics, Science, and Engineering*, 2nd ed., Prentice-Hall, Inc., N.J., U.S.A., 1992.
5. E.A. Volkov, *Numerical Methods*, Mir Publishers, Moscow, 1986.

Paper: MATH.204

Partial Differential Equations and Generalised Functions

Total Lectures: 50

Full Marks: 50

Partial Differential Equations: First order PDE, characteristics of a linear first order PDE, Cauchy's problem. Semi-linear and quasi-linear equations in two-independent variables, general solution, complete integral and singular solution of PDE, solution of on-linear first order PDE by Cauchy's method of characteristics, Charpit's method, Second orderlinear PDE, Adjoint and self-adjoint equations, Reduction of equations in two independent variables to canonical forms, Classifications, well – posed and ill – posed problems.

Elliptic equations: Occurrence of elliptic equation, Fundamental solution of Laplace's equations in two variables, Harmonic function, Characterization of harmonic function by their mean value property, Existence, Uniqueness and Continuous dependence of solutions, Method of separation of variables for the solutions of Laplace's equations, Dirichlet's and Naumann's problems, Green's functions for the Laplace's equations in two dimensions, Solution of Dirichlet's and Naumann's problem for some typical problems like a disc and a sphere, Poisson's general solution.

Hyperbolic equations: Occurrence of Hyperbolic equation, Equation of vibration of a string, Existence, Uniqueness and continuous dependence of the solution on the initial conditions, D'Alembert's solution for the vibration of an infinite string, Domain of dependence, Method of separation of variables, higher dimensional wave equations.

Parabolic equations: Occurrence of parabolic equation, Heat equation - Heat conduction problem for an infinite rod – Heat conduction in a finite rod - existence and uniqueness of the solution.

Generalized Functions: Test functions; Linear functional, Regular and singular generalized functions, Dirac delta function, Translation of a generalized function, Scale of expansion or contraction, differentiation of a generalized functions; Sequence and series of generalized functions, Limit of generalized functions, Fourier transforms of generalized functions; Differential equations of generalized functions.

Texts / References:

1. I. N. Sneddon: *Elements of Partial Differential Equations*, McGraw Hill Publication
2. F. John, *Partial Differential Equations*, Springer-Verlag, New York, 1978.
3. S. Rao, *Introduction to Partial Differential Equations*, 3rd Edition, PHI Learning Private Limited, New Delhi, 2011.
4. J. J. Duistermaat and J. A. C. Kolk, *Distributions Theory and Applications*, Birkhäuser Basel, 2010.
5. I. M. Gel'fand & G. E. Shilov, *Generalized Functions*, Volume 1: Properties and Operations, American Mathematical Society, 2016.
6. V.S. Vladimirov, *Methods of the theory of generalized Functions*, Taylor and Francis, New York, 2002.

Paper: MATH.205 (CBCS)

History of Mathematics

Total Lectures: 50

Full Marks: 50

Ancient Mathematical Sources, Mathematics in Ancient Mesopotamia, The Numeral System and Arithmetic Operations, Geometric and Algebraic Problems, Mathematical Astronomy, Mathematics in Ancient Egypt, Geometry, Assessment of Egyptian Mathematics, Greek Mathematics, The Development of Pure Mathematics, The Pre-Euclidean Period, The Elements, The Three Classical Problems, Geometry in the 3rd Century BCE, Archimedes, Apollonius, Applied Geometry, Later Trends in Geometry and Arithmetic, Greek Trigonometry and Mensuration, Number Theory, Survival and Influence of Greek Mathematics. Mathematics in the Islamic World (8th–15th Century), Origins, Mathematics in the 9th Century, Mathematics in the 10th Century, Omar Khayyam, Islamic Mathematics to the 15th Century. The Foundations of Mathematics : Ancient Greece to the Enlightenment, Arithmetic or Geometry, Being Versus Becoming, Universals, The Axiomatic Method, Number Systems, The Re-examination of Infinity, Calculus Reopens Foundational The Philosophy of Mathematics: Mathematical Platonism, Traditional Platonism, Nontraditional Versions, Mathematical Anti-Platonism, Realistic Anti-Platonism, Nominalism, Logicism, Intuitionism, and Formalism, Mathematical Platonism: For and Against, The Fregean Argument for Platonism, The Epistemological Argument, Against Platonism

Texts / References:

1. Erik Gregersen, *The Britannica Guide to The History of Mathematics*, Britannica.
2. Eleanor Robson, Jacqueline Stedall, *The Oxford Handbook of THE HISTORY OF MATHEMATICS*, Oxford.

Course Structure of M.Sc. in Mathematics

Semester-III				
Course Code	Name of the Course	Full Marks	Total Class Hours	Credit
MATH.301	General Theory of Continuum Mechanics	50	50	5
MATH.302	UNIT-I: Stochastic Process & Statistics UNIT-II:MATLAB Programming (Practical)	25 25	50	5
MATH.303	Special Paper: Operations Research – I / Advanced Algebra – I / Mathematical Modelling In The Biological Environment - I	50	50	5
MATH.304	Special Paper: Operations Research – II / Advanced Algebra – II / Mathematical Modelling In The Biological Environment – II	50	50	5
MATH.305 (CBCS)	Statistics using MATLAB	50	50	4

Paper: MATH.301

General Theory of Continuum Mechanics

Total Lectures: 50

Full Marks: 50

Theory of strain: Configuration of a continuous medium, Continuum hypothesis, Material & Spatial description, Deformation Gradients, Strain measures, Finite strain tensors. Small deformation theory, infinitesimal strain tensor. Relative displacement, Rotation tensor. Interpretation of linear strain tensors. Stretch ratio. Finite strain interpretation. Principal strains. Strain invariant. Cubical dilatation. Saint-Venant's Compatibility equation for strain.

Theory of Stress: Body force. Surface forces. Cauchy's stress principle. Stress vector. State of stress at a point. Stress tensor. Stress vector–stress tensor relationship. Stress equation of equilibrium, Stress tensor symmetry, Stress quadric of Cauchy. Stress transformation laws, Principal stress. Stress invariant. Stress ellipsoid, extremum normal stress and shearing stress.

Conservation Laws: Reynold's Transport Theorem, Conservation of mass, Equation of continuity, Principle of conservation of momentum, Cauchy's first equation of motion, Cauchy's second equation of motion, Conservation of energy, First law of Thermodynamics.

Elasticity: Hook's law, Strain energy function, Isotropic media. Elastic constraints. Moduli of elasticity of isotropic bodies and their relation, elastic constants, Beltrami-Michell compatibility equation, Waves in isotropic elastic media, Navier's equations of motion.

Fluid Mechanics: Kinematics of fluid. Velocity and acceleration in Lagrangian method and Eulerian method, Equation of continuity, Boundary surface. Stream lines and path lines. Constitutive equation of fluid, Euler's equation of motion of Inviscid fluid, Navier Stoke's equation of motion of viscous fluid, Cauchy's integral. Bernoulli's equation. Integration of Euler's equation. Impulsive motion of fluid. Energy equation. Irrotational motion and its physical interpretation. Velocity potential. Flow and circulation. Kelvin's circulation theorem. Kelvin's minimum energy theorem. Motion in two dimensions. Stream function, Complex potential. Source, sink and doublet and their images. Milne-Thompson circle theorem and its applications.

Texts / References:

1. Mechanics of Continua: A. C. Eringen, Wiley.
2. Continuum Mechanics: T.J. Chung, Prentice Hall.
3. Continuum Mechanics: Schaum's out line of Theory and Problems of Continuum Mechanics: Gedrg R. Mase, Mc Graw Hill.
4. Continuum Mechanics: D. S. Chandra sekharaiiah and L. Debnath, Academic Press.
5. Mathematical Theory of Continuum Mechanics: R.N. Chatterjee, Narosa Publishing House.
6. Continuum Mechanics: A.J.M. Spencer, Long man.
7. Mathematical theory of Elasticity: I. S. Sokolnikoff, Tata Mc Grow Hill Co.

Paper: MATH.302

Unit-I: Stochastic Process & Statistics

Total Lectures: 25

Full Marks: 25

Stochastic Process : Concept and classification, Definition of a chain, Markov property, Markov chain with finite and countable state space, Classification of states, Recurrent and Transient states, Null recurrent and Positive recurrent states, Mean Ergodic Theorem, Random walk and Gambler's ruin chain, Branching chain, Birth – and – Death chain, Probability of ultimate ruin in a Gambler's ruin chain, Expected duration of a Gambler's ruin chain, Markov processes in continuous time, Birth and Death process, Pure Birth process, Poisson process.

Partial and multiple correlation coefficients, Principle component analysis, Discriminant analysis, Cluster analysis, Advanced theory of linear estimation.

Texts / References :

1. Samuel Karlin and Howard E. Taylor, *A First Course in Stochastic Process*, Academic Press.
2. Hoel, Port and Stone, *Introduction to Stochastic Process*, Waveland Press.
3. K.L.Chung: *Elementary Probability Theory and Stochastic Process*
4. A.M. Goon, M.K. Gupta & B. Dasgupta, *Fundamentals of Statistics*, Vol. 1 & 2, Calcutta: The World Press Private Ltd., 1968.
5. J. Medhi, *Stochastic Process*, New Age International Publisher, 2ed, 1984.
6. S.K.Srinivasan and K.M. Mehata: *Stochastic Process*
7. Suddhendu Biswas, G. L. Sriwastav, *Mathematical Statistics: A Textbook*, Narosa, 2011

UNIT-II: MATLAB Programming (Practical)

Total Lectures: 25

Full Marks: 25

Programming in MATLAB: The MATLAB workspace, data types, variables, assignment statements, arrays, sets, matrices, string, time, date, cell arrays and structures, introduction to M – file scripts, input and output functions, conditional control statements, loop control statements, break, continue and return statements.

Working with matrix: Generating matrix, Concatenation, Deleting rows and columns. Symmetric matrix, matrix multiplication, Test the matrix for singularity, magic matrix. Matrix analysis using function: norm, normest, rank, det, trace, null, orth, rref, subspace, inv, expm, logm, sqrtm, funm.

Array: Addition, Subtraction, Element-by-element multiplication, Element-by-element division, Element-by-element left division, Element-by-element power. Multi-dimensional Arrays, Cell Arrays, Characters and Text in array,

Graph Plotting: Plotting Process, Creating a Graph, Graph Components, Figure Tools, Arranging Graphs Within a Figure, Choosing a Type of Graph to Plot, Editing Plots, Plotting Two Variables with Plotting Tools, Changing the Appearance of Lines and Markers, Adding More Data to the Graph, Changing the Type of Graph, Modifying the Graph Data Source, Annotating Graphs for Presentation, Exporting the Graph.

Using Basic Plotting Functions: Creating a Plot, Plotting Multiple Data Sets in One Graph, Specifying Line Styles and Colors, Plotting Lines and Markers, Graphing Imaginary and Complex Data, Adding Plots to an Existing Graph, Figure Windows, Displaying Multiple Plots in One Figure, Controlling the Axes , Adding Axis Labels and Titles, Saving Figures.

Programming: Conditional Control – if, else, switch, Loop Control – for, while, continue, break, Error Control – try, catch, Program Termination – return.

Scripts and Functions: Scripts, Functions, Types of Functions, Global Variables, Passing String Arguments to Functions, The eval Function, Function Handles, Function Functions, Vectorization, Pre allocation.

Linear Algebra: Systems of Linear Equations, Inverses and Determinants, Factorizations, Powers and Exponentials, Eigen values, Singular Values.

Polynomials: Polynomial functions in the MATLAB® environment, Representing Polynomials, Evaluating Polynomials, Roots, Derivatives, Convolution, Partial Fraction Expansions, Polynomial Curve Fitting, Characteristic Polynomials.

Texts / References:

1. Gilat A. MATLAB: an Introduction with Applications. New York: Wiley; 2008.
2. Palm III WJ. Introduction to MATLAB for Engineers. New York: McGraw-Hill; 2011.
3. Chapman SJ. MATLAB programming with applications for engineers. Cengage Learning; 2012.
4. Lopez C. MATLAB programming for numerical analysis. Apress; 2014.

Paper: MATH.303A (Special Paper)

Operational Research-I

Total Lectures: 50

Full Marks: 50

Revised simplex method (with and without artificial variable). Modified dual simplex.

Large Scale Linear Programming: Decomposition Principle of Dantzig and Wolf.

Parametric and post-optimal analysis: Change in the objective function. Change in the requirement vector, Addition and deletion of variables, Addition and deletion of constraints., Parametric analysis bound algorithm.

Convex and concave functions: Basic properties and some fundamental theorems of convex/concave functions, Differentiable convex and concave functions.

Of cost and requirement vector.

Integer Programming: Gomory's cutting plane algorithm, Gomory's mixed integer programming algorithm, Branch and

Classical optimization techniques: Single variable optimization, multivariate optimization(with no constraint, with equality constraints and with equality-inequality constraints).

Quadratic Programming: Wolfe's modified simplex method, Beale's method, Convex programming.

Unconstrained optimization: Gradient methods, Steepest descent method, conjugate gradient method, Newton's method, Quasi-Newton's method, Daviddon-Fletcher-Powell method

Texts / References:

1. Edwin K. P. Chang & S. Zak. *An Introduction to Optimization*. John Wiley & Sons Inc., 2004.
2. S. S. Rao. *Engineering optimization: theory and practice*. John Wiley & Sons, 2009.
3. Taha, Hamdy A. *Operations research: An introduction*. Pearson Education India, 2004.
4. Belegundu, Ashok D., and Tirupathi R. Chandrupatla. *Optimization concepts and applications in engineering*. Cambridge University Press, 2011.
5. Sharma, S. D. *Operations Research*, KedarNath Ram Nath & Co., Meerut.

Paper : MATH.303B (Special Paper)

Advanced Algebra – I

Total Lectures : 50

Full Marks : 50

Module, Submodule, Quotient module, Homomorphism and isomorphism of modules, Exact sequences, short and split exact sequences, tensor product of modules, definition and existence, universal mapping property, further properties of the tensor product of modules, projective modules, injective modules, flat Modules, direct sum of projective modules, direct product of injective modules.

Nilradical and Jacobson radical, Chinese remainder theorem, extension and contraction of ideals, rings and modules of fractions, local ring, characterizations of local ring, localization, Extended and contracted ideals in rings of fractions, Noetherian rings, Artinian rings, Hilbert Basis Theorem, Cohen's Theorem, Primary Decomposition in Noetherian Rings.

Texts / References :

1. Dummit, D.S. and Foote, R.M., *Abstract Algebra, Second Edition*, John Wiley & Sons, Inc., 1999.
2. Atiyah, M., MacDonald, I.G., *Introduction to Commutative Algebra*, Addison-Wesley, 1969.
3. Lang, S., *Algebra*, Addison-Wesley, 1993.
4. Lam, T.Y., *A First Course in Non-Commutative Rings*, Springer Verlag.
5. Hungerford, T.W., *Algebra*, Springer.
6. Jacobson, N., *Basic Algebra, II*, Hindusthan Publishing Corporation, India.
7. Gopalakrishnan, N. S., *Commutative Algebra*, 2nd edition, Orient Blackswan, 2015.
8. Balwant Singh, *Basic Commutative Algebra*, World Scientific Publishing Co Pte Ltd,

Paper : MATH.303C (Special Paper)

Mathematical Modelling in the Biological Environment - I

Total Lectures : 50

Full Marks : 50

Single Species Population Models

Introduction, Fundamental Concepts, Malthus Model of Exponential Growth, Formulation of the models of (i) continuous-time version and (ii) discrete-time version, solution and interpretation, limitations, applications.

Logistic Growth Model, formulation of the model, solution and interpretation, limitations, applications. Harvesting of the population at a constant rate, solution and interpretation, Biological Overexploitation, catch-per-unit-effort hypothesis, minimum viable population level. Yield-Effort Curve for a pure compensation model, depensation model and critical depensation model, irreversible extinction process.

Texts / References :

1. Thomas Malthus (1798) An Essay on the Principle of Population, J. Jonson, London.
2. P. F. Verhulst (1938) Notice sur la que la populationsuit dans son Accroissement, Correspondence Mathematique et Physique, **10**:113-121.
3. A. J. Lotka (1925) Elements of Physical Biology, Williams and Wilkins, Baltimore, Md.
4. V. Volterra (1926) Variazioni e fluttazioni del numerod'individui in specie animaliconviventi, Mem. Accad. Lincei, **6**:31-113.
5. M. Martcheva (2015) An Introduction to Mathematical Epidemiology, Volume 61, New York: Springer.
6. F. Brauer and C. Castillo-Chavez (2012) Mathematical Models in Population Biology and Epidemiology (Second Edition), New York: Springer.
7. M. J. Keeling and P. Rohani (2011) Modelling Infectious Diseases in Humans and Animals, Princeton University Press.
8. C. W. Clark (1990) Mathematical Bioeconomics: The Optima/ Management of Renewable Resources (Second Edition), John Wiley & Sons, New York.
9. C. W. Clark (1985) Bloeconomic Modelling and Fisheries Management, John Wiley & Sons, New York.
10. M. B. Schaefer (1954) Some aspects of the dynamics of populations importantto the management of commercial marine fisheries, Bulletin of Inter-American Tropical Tuna Commission, **1**:25-26.
11. H. S. Gordon (1954) Economic theory of a common property resource: the fishery, Journalof Political Economy, **62**: 124-142.
12. R. J. H. Beverton and S.J. Holt (1957) on the dynamics of Exploited Fish Populations, Ministry of Agriculture, Fisheries investigations series **2(19)**, London.
13. P. A. Samuelson (1976) The Economics of forestry in an evolving society, Economic Inquiry **XIV**, pp.466-492.
14. F. Y. M. Wan (1985) Ordered site access and optimal forest rotation, Studies in Applied Mathematics, **73**: 155-175.
15. M. Faustmann (1968) On the determination of the value which forest lands and immature stands pose for forestry, in M. Gane (Ed.) Martin Faustmann and the evolution of discounted cash flow, oxford institute Paper No. **42**.
16. L. S. Pontryagin , V. S. Boltyanskii, R. V. Gamkreiidge and E. F. Mischenko (1962) The Mathematical Theory of Optimal Processes, Wiley-Interscience, New York.

Paper: MATH.304A (Special Paper)

Operational Research-II

Total Lectures: 50

Full Marks: 50

Inventory control: Deterministic Inventory control including price breaks and Multi-item inventory models with constraints. Probabilistic inventory control (with and without lead time), Dynamic inventory models. Basic concept of Supply chain management and two echelon supply chain model.

Replacement and Maintenance Models: Introduction, Failure Mechanism of items, Replacement of items deteriorated with time, Replacement policy for equipments when value of money changes with constant rate during the period, Replacement of items that fail completely— individual replacement policy and group replacement policy, Other replacement problems — staffing problem, equipment renewal problem.

Queueing Theory: Basic features of queueing systems, operating characteristics of a queueing system, arrival and departure (birth & death) distributions, inter-arrival and service times distributions, transient, steady state conditions in queueing process. Poisson queueing models- $M/M/1$, $M/M/C$ for finite and infinite queue length. Machine-Maintenance problem (steady state), power supply model, Non-Poisson queueing models- $M/E_k/1$, $M/G/1$, mixed queueing model $M/D/1$, cost models in queueing system.

Sequencing: Sequencing problems, Solution of sequencing problems, Processing of n jobs through two machines, Processing of n jobs through three machines, Optimal solutions, Processing of two jobs through m machines, Graphical method of solution, Processing of n jobs through m machines.

Simulation: Introduction, Steps of simulation process, Advantages and disadvantages of simulation, Stochastic simulation and random numbers— Monte Carlo simulation, Random number generation, Simulation of inventory problems, Simulation of queueing problems, Role of computers in simulation, Applications of simulations.

Theory of Games: Preliminary concept of continuous game, Bi-matrix games, Nash equilibrium, and solution of bi-matrix games through quadratic programming (relation with nonlinear programming).

Texts / References:

1. Taha, Hamdy A. Operations research: An introduction. Pearson Education India, 2004.
2. Sharma, S. D. Operations Research, KedarNath Ram Nath & Co., Meerut.
3. Sharma J.K. Operations Research: theory and application, Macmillan Publishers, 2006.
4. Hillier, F.S., 2012. Introduction to operations research. Tata McGraw-Hill Education.

Paper: MATH.304B (Special Paper)

Advanced Algebra – II

Total Lectures : 50

Full Marks : 50

Universal algebra, examples, subalgebra, subuniverse, congruences and quotient algebra. homomorphisms of universal algebra, kernel of a homomorphism. First Isomorphism Theorem, Second Isomorphism Theorem, Third Isomorphism Theorem. semigroup of endomorphisms of a universal algebra. Birkhoff's theorem. Special faithful representation of universal algebras into semigroups, Theorem of Cohn-Rebane.

Free algebra. direct and subdirect product of universal algebras. subdirectly irreducible universal algebras. Class operators. Turski's theorem, identities. Term algebra, universal mapping property, k -free universal Algebra, variety, Birkhoff's theorem on variety, Macev's conditions.

Group representations, group-rings, Maschke's theorem, character of a representation, regular representations, orthogonality relations, Burnside two-prime theorem.

Texts / References :

1. Stanley Burries and H. P. Sankappanavan; *Universal Algebra*; Springer-Verlag.
2. Curtis, C.W., Reiner, I., *Representation Theory of Finite Groups and Associated Algebras*, Wiley-Interscience, NY.
3. Benjamin Steinberg, *Representation Theory of Finite groups, An Introductory Approach*, Springer, 2012.

Paper: MATH.304C (Special Paper)

Mathematical Modelling in the Biological Environment - II

Total Lectures : 50

Full Marks : 50

Two-Species Population Models

Introduction, Objectives, Types of interactions between two species, Prey-predator Model, Formulation, analytic solution and interpretation, stability, examples.

Lotka-Volterra model for two competing species, formulation, solution and geometrical interpretation, limitations, applications.

Autonomous system of two first order equations, trajectory (orbit) of the system, equilibrium, almost linear system, stability of the system.

Epidemics

Introduction, Objectives, Basic Definitions and Fundamental Concepts. Simple Epidemic Model, Formulation, Solution and Interpretation, Limitations. General Epidemic Model, Formulation, Solution and Interpretation, Kermack-McKendrick Threshold Theorem, Limitations. Recurrent Epidemic Model, Solution and Interpretation, Limitations. Recurrent Epidemic Model, Formulation, Solution and Interpretation, Limitations. Recurrent Epidemic Model with Undamped Waves.

Texts / References : Same as Paper MATH.303C

Paper: MATH.305 (CBCS)

Statistical Methods using MATLAB

Total Lectures: 50

Full Marks: 50

This course provides hands – on experience with performing data analysis with MATLAB.

Introductory MATLAB: Why MATLAB, familiar with MATLAB windows, basic operations, data types, rules about variable names, predefined variables, built –in- functions, arrays, input and output, arithmetic operator, logical operator, relational operator, if statement, switch statement, loop statement and script file.

Basic statistical investigation of a data set: Plotting, Central Tendency, Spread, Shape, Correlations, Grouped data

Investigation of different probability distributions: Fitting the probability distributions to a data set

Predictive modelling by fitting linear and nonlinear models to a data set: Linear regression models, fitting linear models to data, evaluating the fit

Random numbers and simulation: Generating numbers from standard distributions, generating numbers from arbitrary distributions and controlling the random number system

Texts / References:

1. Gilat A. MATLAB: an Introduction with Applications. New York: Wiley; 2008.
2. Palm III WJ. Introduction to MATLAB for Engineers. New York: McGraw-Hill; 2011.
3. Chapman SJ. MATLAB programming with applications for engineers. Cengage Learning; 2012.
4. Lopez C. MATLAB programming for numerical analysis. Apress; 2014.

Course Structure of M.Sc. in Mathematics

Semester-IV				
Course Code	Name of the Course	Full Marks	Total Class Hours	Credit
MATH.401	Functional Analysis	50	50	5
MATH.402	Mathematical Methods	50	50	5
MATH.403	Special Paper: Operations Research – III / Advanced Algebra – III / Computational Fluid Dynamics – III / Mathematical Modelling in the Biological Environment – III	50	50	5
MATH.404	Special Paper: Operations Research – IV / Advanced Algebra – IV / Computational Fluid Dynamics – IV / Mathematical Modelling in the Biological Environment – IV	50	50	5
MATH.405	Social Outreach Programme DISSERTATION PROJECT WORK	10 40	50	5

Paper: MATH.401

Functional Analysis

Total Lectures: 50

Full Marks: 50

Normed Linear Space (NLS), Banach Space and its basic properties, Equivalent norms, Finite dimensional NLS, Heine – Borel Theorem, Riesz Lemma and best approximation property, Quotient space of normed linear space and its completeness, Bounded linear operators and its basic properties, Space of bounded linear operators, Uniform boundedness principle and some applications, Open Mapping Theorem, Closed Graph Theorem, Linear functional, Hahn-Banach Theorem, Dual Space, Inner Product Space (IPS), Hilbert Space, Orthonormal basis, Complete orthonormal basis, Cauchy-Schwarz inequality, Parallelogram law, Bessel's inequality, Parseval's identity, Orthogonal complement and Projection Theorem, Inner product is a continuous operator, Relation between IPS and NLS, Strong and weak convergence of sequence of operators, Reflexivity of Hilbert Space, Riesz Representation Theorem for bounded linear functional on a Hilbert Space, Adjoint of an operator on a Hilbert Space, Definition of self-adjoint operator, Normal, Unitary and Positive operators, Related simple theorems.

Texts / References:

1. B.V. Limaye, Functional Analysis, 2nd Edition, New Age International Private Limited New Delhi, 2014.
2. J. B. Conway, A Course in Functional Analysis, 2nd Edition, Springer-Verlag New York, 1985.
3. E. Kreyzig, Introduction to Functional Analysis with Applications, John Wiley & Sons, New York, 1989.
4. A. Taylor and D. Lay, Introduction to Functional Analysis, Wiley, New York, 1980.

Paper: MATH.402

Mathematical Methods

Total Lectures: 50

Full Marks: 50

Integral Transforms: Laplace Transform, Properties of Laplace transform, Inversion by analytic method and Bromwich path, Convolution theorem, Lerch's theorem, Applications to differential equations.

Fourier Transform and its properties, Inversion formula, Convolution theorem, Parseval's relation, Applications.

Wavelet Transforms: Time-frequency analysis, Multi-resolution analysis, Spline wavelets, scaling function, Short-time Fourier transforms, Wavelet series, orthogonal wavelets, Applications to signal and image processing.

Integral Equations: Reduction of differential equations to integral equations, Existence and uniqueness, Types of Integral equations - Fredholm and Volterra, Solution processes - successive substitutions, successive approximations, degenerate kernel, resolvent Kernel, symmetric kernel, singular integral equation and Abel's integral equation.

Operator Equations: Hilbert space, Fourier expansion, Reisz Fisher theorem, adjoint, self-adjoint, Eigen value problems, Spectral theorem for compact self – adjoint operations, integral equations with Hilbert – Schmidt kernel, solubility of operator equations.

Texts / References:

1. I.N.Sneddon: Fourier Transforms, McGraw - Hill.
2. S.G.Mikhlin: Integral Equations, Pergamon Press
3. P. P. G. Dyke, An Introduction to Laplace Transform and Fourier Series, Springer 2005.
4. R.P. Kanwal: Linear Integral Equations; Theory & Techniques, Academic Press, New York, 1971.
5. W. V. Lovitt, Linear Integral Equations, Dover Publications, 1950.
6. R.V.Churchill: Operational Methods

Paper: MATH.403A (Special Paper)

Operational Research-III

Total Lectures: 50

Full Marks: 50

Optimization: Nature and Scope of optimization, Optimality criterion of linear programming Application of Farkas's theorem, Existence theorem for linear systems, Theorems of the alternatives, Slater's theorem of alternatives, Motzkin theorem of alternatives, Saddle point optimality criteria without differentiability, Slater's and Karlin's constraint qualifications and their equivalence, strict constraints qualification, Kuhn-Tucker's saddle point necessary optimality theorem, Fritz-John saddle point optimality theorem, Optimality criterion with differentiability and Convexity, Kuhn-Tucker's sufficient optimality theorem, Fritz-John stationary point optimality theorem, Duality in non-linear programming, Weak duality theorem, Wolfe's duality theorem, Duality for quadratic programming.

One-dimensional optimization: Function comparison method, Fibonacci and Golden section methods for unimodal functions

Constrained optimization: Methods of feasible direction and cutting hyperplane method.

Geometric Programming: Geometric programming (both unconstrained and constrained) with positive and negative degree of difficulty.

Dynamic programming: Basic features of dynamic programming problems, Bellman's principle of optimality, multistage decision process-forward and backward recursive approaches, Dynamic programming approach for solving (i) linear and nonlinear programming problems, (ii) routing problem, (iii) reliability optimization problem, (iv) inventory control problem, (v) cargo loading problem (vi) Allocation problem

Goal Programming: Introduction, Difference between LP and GP approach, Concept of Goal Programming, Graphical solution-method of Goal Programming, Modified simplex method of Goal Programming.

Texts / References:

1. Mokhtar S. Bazaraa, Hanif D. Sherali and C.M. Shetty, Nonlinear Programming: Theory and Algorithms, John Wiley & Sons, 2006.
2. Olvi L. Mangasarian, Nonlinear Programming, Society for Industrial and Applied Mathematics, 1994.
3. S.S. Rao, Engineering Optimization: Theory and Practice, John Wiley & Sons, 1996.
4. Frederick S. Hillier and Gerald J. Lieberman, Introduction to Operations Research, McGraw-Hill, 2010.

Paper : MATH.403B (Special Paper)

Advanced Algebra – III

Total Lectures : 50

Full Marks : 50

Noetherian module and Artinian module, composition series, simple and semisimple modules, Semisimple ring, Group ring, Characterizations of semisimple ring, Wedderburn-Artin theorem on semisimple ring, simple ring, characterization of Artinian simple ring.

Prime and semiprime ideal, m-system and n-system, prime radical, prime rings, semiprime ring, ring derivation on prime and semiprime rings.

Jacobson semisimple ring, relation between Jacobson semisimple ring and semisimple ring, regular ring, idempotents in regular rings, lifting of idempotents, relation among semisimple ring, regular ring and Jacobson semisimple ring, primitive ring, structure of primitive ring, Jacobson-Chevalley density theorem, Wedderburn-Artin theorem on primitive ring.

Subdirect sum of rings, Representation of a ring as a subdirect sum of rings, Subdirectly irreducible ring, Birkhoff theorem on subdirectly irreducible ring, Subdirectly irreducible Boolean ring, Stone Representation Theorem.

Texts / References :

1. T.Y. Lam; *Noncommutative Rings*; Springer-Verlag.
2. I.N. Herstein; *Noncommutative rings*; Carus monographs of AMS, 1968.
3. N. Jacobson; *Structure of Rings*; AMS.
4. L.H. Rowen; *Ring theory (student edition)*; Academic Press, 1991.
5. T.W. Hungerford; *Algebra*; Springer, 1980.

Paper : MATH.403C (Special Paper)

Mathematical Modelling in the Biological Environment - III

Total Lectures : 50

Full Marks : 50

Economic Methods of Renewable Resource Harvesting

The open access fishery, Gordon's model and principal results, Opportunity cost, Externality, Economic Overfishing, Cobb-Douglas Production Function, constant-return-to-scale, decreasing-return-to-scale and increasing-return-scale-properties.

Interest and discount rate, concept of the *present value (PV)* of a payment due to be paid after arbitrary number of years. The harvesting model considered by the biologist M.B. Schaefer, biological equilibrium and economic overfishing, the optimal harvest policy in the Schaefer model.

Growth and Aging in the Forestry Models

Introduction, Forestry Management: The Faustmann Model, Faustmann Formula, average annual economic yield, response to change in demand. Chris Rorres Model, non-harvest vector, growth matrix, harvest vector, sustainable harvesting policy, optimal sustainable yield, illustrations, examples. A matrix model for the management of a height structured forest, growth parameters and growth matrix, non-harvest vector, harvest vector, sustainable harvesting policy, optimal sustainable yield, illustrations and examples. Beverton-Holt fisheries model, natural mortality rate and fishing mortality rate, knife-edge selectivity of the fishing gear, relation between the total annual yield from all cohorts and the total lifetime yield of a single cohort., dynamic optimization in the Beverton-Holt model, application of Pontryagin's Maximum Principle.

Texts / References : Same as Paper MATH.303C

Paper: MATH.404A (Special Paper)

Operational Research-IV

Total Lectures: 50

Full Marks: 50

Optimal Control Theory: Introduction to Optimal Control: Control and Optimal Control, Examples, The Basic Optimal Control Problem, Variational Calculus; Optimal Control with Unbounded Continuous Controls: The Hamiltonian, Extension to Higher Order Systems; Bang-Bang Control, Pontryagin's Principle, Switching Curves, Transversality Conditions; Applications of Optimal Control in Economic Growth, Exploited Populations, and Advertising Policies.

Reliability: Definition of reliability, Measures of reliability, system reliability, System Failure rate, reliability of different systems, like series, parallel, series parallel, parallel-series, k-out-of-n, etc., MTBF, MTTF, optimization using reliability, reliability and quality control comparison, reduction of life cycle with reliability, maintainability, availability, Effect of age, stress, and mission time on reliability. Idea of reliability optimization.

Information Theory: Introduction, Communication Processes— memory less channel, the channel matrix, Probability relation in a channel, noiseless channel. Measure of information- Properties of Entropy function, Measure of other information quantities — marginal and joint entropies, conditional entropies, expected mutual information, Axiom for an Entropy function, properties of Entropy function. Channel capacity, efficiency and redundancy. Encoding-Objectives of Encoding. Shannon-Fano Encoding Procedure, Necessary and sufficient Condition for Noiseless Encoding.

Project Network: PERT and CPM: Introduction, Basic difference between PERT and CPM, Steps of PERT/CPM Techniques, PERT/CPM Network components and precedence relationships, Critical path analysis, Probability in PERT analysis, Project Time-Cost, Trade-off, Updating of the project, Resource allocation — resource smoothing and resource levelling.

Stochastic Programming: Chance constraint programming technique.

Multi-objective Non-linear Programming: Introductory concept and solution procedure.

Texts / References:

1. Swarup, K., Gupta, P.K and Man Mohan, Operation Research, Sultan Chand & Sons.
2. Sharma, J.K Operation Research – Theory and Application, Macmillan.
3. Gupta, P.K. and Hira, D.S., Operation Research, S. Chand & Co. Ltd.
4. Taha H.A., Operation Research – an Introduction, PHI.
5. Bronson, R. and Naadimuthu. G., Theory and problems of Operations Research, Schuam's Outline Series, MGH.

Paper: MATH.404B (Special Paper)

Advanced Algebra –IV

Total Lectures : 50

Full Marks : 50

Category, examples, dual category, special morphisms, monic and epic, retraction and coretraction, Functor, forgetful functor, faithful functor, product category, bifunctor, natural transformation, representable functor, embedding, Yoneda's lemma and its applications, Adjoint functor, Initial object, terminal object, zero object, Limit and colimit, Pull back diagram and push out diagram.

Lattice, sublattice, generators of a sublattice, ideal, dual ideal, homomorphisms and isomorphisms, congruences, distributive lattice, characterization theorems and representation theorems on distributive lattice, complete lattice, structure of the lattice of congruences of lattices, semi modular lattices, characterization theorems and representation theorems, Stone's representation theorem on Boolean Algebra, algebraic lattice – characterization theorems and representation theorems, correspondence between ideals and congruences of lattices, Boolean Algebra and generalized Boolean Algebra.

Texts / References :

1. *S. MacLane; Category theory for working mathematician; Springer-Verlag.*
2. *B. Mitchel; Theory of categories; Academic Press (1969).*
3. *G. Szasz – Lattice Theory*
4. *G. Birkhoff – Lattice Theory*
5. *B. H. Arnold – Logic and Boolean Algebra*
6. *D. E. Rutherford – Lattice Theory*

Paper: MATH.404C (Special Paper)

Mathematical Modelling in the Biological Environment - IV

Total Lectures : 50

Full Marks : 50

Beverton-Holt Fisheries Model

Beverton-Holt fisheries model, natural mortality rate and fishing mortality rate, knife-edge selectivity of the fishing gear, relation between the total annual yield from all cohorts and the total life-time yield of a single cohort., dynamic optimization in the Beverton-Holt model, application of Pontryagin's Maximum Principle.

Multiple Species Model

A model of combined harvesting of two ecologically independent fish populations assuming that fishing costs are proportional to harvesting effort, net economic revenue, bionomic equilibrium, ultimate extinction of the species, optimal harvest policy, optimal equilibrium solution, singular control. Gause's model of inter-specific competition, biotic potentials, functional response, competitive exclusion. Harvesting competing populations, Clark's investigation.

Texts / References : Same as Paper MATH.303C

Paper: MATH.405

Unit-I: Social Outreach Programme

Full Marks: 10

Social outreach programme will be done according to the decision of the Department in every year.

Unit-II: Dissertation Project Work

Full Marks: 40

Project Work of each student will be evaluated by the concerned internal teacher/supervisor and one External Examiner. Marks distribution of Project Work is as follows: Project Work-20, Presentation-10 & VivaVoce-10.